

EXPERIMENTAL INVESTIGATION AND NUMERICAL ANALYSIS ON INFLUENCE OF FOUNDATION EXCITATION ON THE DYNAMICS OF THE ROTOR SYSTEM

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ABSTRACT

Rotor system of modern aero-engine together with the case mounted on the wing represents a uniform system that is supposed to be considered as a rotor-bearings-foundation structure. Nowadays rotating machinery such as modern aircraft engines usually designed, marketed and sold for the most part based on analytical and numerical predictions. In such a way methods to incorporate the foundation effect in rotordynamic calculations are very important. For investigation purposes a rotor-foundation test rig, which can simulate the aero-engine's typical operating condition such as wing vibration and hard landing, was built to study the influence of foundation behavior on the dynamic characteristics of rotor system. To predict natural frequencies for the full system simplified models based on FEM approach were created. Moreover, simple numerical model was created to study influence of foundation kinematic excitation on behavior of rotor disk orbit. Furthermore simulation results were compared with experimental to understand influence of main parameters which define foundation vibration on rotor deviation from normal operation condition. Obtained numerical and experimental results can help to understand principles of rotating structure-foundation interaction when the later one subjected to excitation and could be further used for improving of more complicated models for design and enhancement of aircraft engines.

1. INTRODUCTION

Nowadays rotating machinery such as modern aircraft engines usually designed, marketed and sold for the most part based on analytical and numerical predictions. As a rule of thumb, rotor system of modern aero-engine together with the case mounted on the wing represents a uniform system that is supposed to be considered as a rotor-foundation structure. Meanwhile this system is subjected to complex loads. Those from them which are connected with the rotating structure could be designated as internal excitation sources, while the others such as airborne loads, landing loads, foreign object damage and etc., acting first on foundation structure could be called external excitation sources. Existence of multiple vibration sources increases the risk of natural frequencies excitation for the full rotor-foundation system which in turn can damage the engine and its components.

Generally, in the field of rotordynamics there are two main approaches for foundation structure modeling: discrete and continuous. The first one includes simplified presentation of the foundation structure using lumped masses, while the

second one considers foundation as a multi-degree model. Simplified support structure models could be found in works of Lund^[1, 2] and Gunter^[3] about problems of instability for Laval-Jeffcott rotor, or in papers of Gunter^[4], Chen^[5], Nicholas et al.^[6-7] focused on modeling of rotor-bearing systems for real gas turbine machines. Kang et al^[8] described in details how to build models with lumped mass foundation. Qingkai Han et al.^[9] used simplified two-mass single-degree of freedom model to predict the behavior of the single disk rotor mounted on elastically supported foundation base. It could be said that simplified foundation structure modeling is quite advantageous due to its simplicity, fast assembling and convenient applicability in standard rotordynamic software. At the same time if several foundation modes are presented in the operating range of the machine, this approach might be not informative and could not fully describe the real dynamic behavior of the rotor-bearing-foundation system.

Thus, recently there is a tendency among engineers all over the world to use foundation models with multi-degree of freedoms. Examples of multi-degree of freedom foundation models could be found in works of Vasquez and Barrett^[10, 11], Edwards et al.^[12-14], Cavalca et al.^[15-17], Feng and Hahn^[18-21], Pennacchi et al.^[22, 23]. Experimental approaches to include foundation structure effect in rotordynamics model were described by Gash^[24], Stephenson and Rouch^[25], Murphy and Vance^[26]. Fundamental works about rotor-foundation systems modeling was made by Kramer^[27], Kostyk et al.^[28-30], Savinov^[31], Shulzhenko et al.^[32, 33] in the field of heavy power machinery. Practical cases of incorporation elasticities in rotordynamic calculations could be found in [34-36].

Previously authors^[37] highlighted that currently there is a tendency among aerospace engineers for improved calculation and development of reliable rotor-foundation structure models, in addition the paper gives a detailed review on recent methods for foundation structure modeling and testing. It was also revealed that even with usage of advanced sophisticated computer programs, it is still difficult and time-consuming to establish reliable rotor-foundation system model using finite element methods. Nevertheless the most common method to analyze the response of the system from the dynamic loads on the stage of design is still a mathematic modeling.

Situation when aircraft wing is subjected to excitation from airborne loads during the flight or from impact loads when landing could be considered as problem of kinematic excitation, which was widely studied in general dynamic of the machine in the field of transport^[38, 39] on the base of simplified models. In the field of ground power machinery problem of foundation kinematic excitation was studied by

Shatochin and Zimmerman on example of steam turbine rotor-train [40, 41]. Detailed mathematical models for rotor-foundation structure were developed by Chang-Sheng Zhu [42, 43] for rotor on magnetic bearings subjected to base excitation and by Guang-Chi Ying [44, 45] in the field of light turbocharger rotor in order to investigate differences in dynamic behavior for the system modeled with foundation and without it.

Keen interest to dynamics of rotating machines subjected to foundation base excitation can be noted in the field of earthquake engineering for power plants where investigation of influence from seismic excitation was studied [46-48]. Seismic analysis of rotating machines is significantly different from structural seismic analysis because of existence of gyroscopic forces for rotors and considerable Coriolis force, emerging on rotors due to angular motions of foundation bases, in addition to presence of supports with nonlinear elastic and damping properties [49, 50]. Since aircraft engine is usually attached rigidly to the wing, excitation of latter from airborne loads or when aircraft moves on rough surface before takeoff or after landing can be transmitted directly to the rotating parts, what in turn can lead to its unsteady motion and damage important components. In such a way investigation of vibration sources which can lead to damage of the machine and its parts is an important practical problem. On technical side that means creation of numerical models which can predict the trajectory of movement and approximate maximum deviation from normal operation condition after applied foundation excitation. To fulfill this requirement numerical model should reflect the behavior of the rotor which would be close to what could happened in real machine and provide an opportunity to analyze rotor orbits. At the same time model should not be complicated too much in order to save time for analysis and be easy to learn for use by engineers from different industries where gas turbine engines are applied.

However number of works in the field of rotating machines focused on observation of rotor orbits when rotor-foundation structure is subjected to kinematic excitation is quite limited. In such a way the aim of current paper is to create simple numerical models and compare its results with experimental in order to evaluate the influence of the main factors such as frequency and amplitude when foundation structure is excited.

2. THEORY OF FOUNDATION BASE EXCITATION

One of the main issues in the field of foundation structure analysis is an objective prediction of rotating machine safe and adequate behavior when foundation structure is subjected to excitation. For basic understanding of the phenomena it is convenient to consider a single mass model on foundation base which is subjected to excitation via displacement or acceleration, Fig. 1. The system mounted on foundation consist of mass, dashpot with damping c and spring with stiffness k . Undamped natural frequency of the system could be written as:

$$\omega = \sqrt{k/m} \quad (1)$$

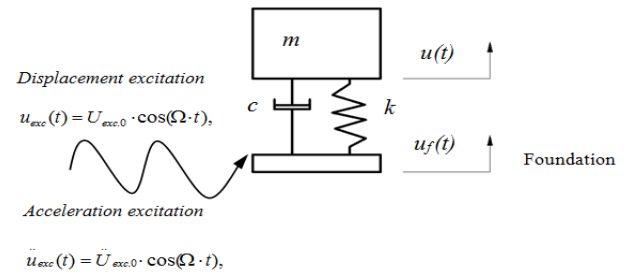


Fig. 1 Single mass model on foundation base: $u_{exc}(t)$ - is vertical displacement of the base, $U_{exc.0}$ - is amplitude of the foundation base movement, $\ddot{u}_{exc}(t)$ - is vertical acceleration of the base, $\ddot{U}_{exc.0}$ - is amplitude of foundation base acceleration, Ω - is vibration frequency

For prediction of the vertical displacement $u(t)$ of single mass m mounted on excited foundation the differential equation of the system for displacement excitation could be presented as:

$$m\ddot{u} + c(\dot{u} - \dot{u}_f) + k(u - u_f) = 0 \quad (2)$$

, which after rearranging could be written in more clear form as:

$$m\ddot{u} + c\dot{u} + ku = ku_f + c\dot{u}_f \quad (3)$$

, where the right side of the equation (3) could be noted as external excitation force:

$$F(t) = ku_f + c\dot{u}_f \quad (4)$$

At the same manner the differential equation for the case of base acceleration excitation could be written as:

$$m(\ddot{u} - \ddot{u}_f) + c(\dot{u} - \dot{u}_f) + k(u - u_f) = -m\ddot{u}_f \quad (5)$$

In both cases the maximum displacement and maximum absolute vertical acceleration of the single mass m could be obtained as:

$$\frac{u_{max}}{U_{exc.0}} = \frac{\ddot{u}_{max}}{\ddot{U}_{exc.0}} = \sqrt{\frac{1 + (2\xi\beta)^2}{(1 - \beta^2)^2 + (2\xi\beta)^2}} \quad (6)$$

, where β is a frequency ratio: $\beta = \Omega/\omega$ and ξ is damping ratio $\xi = \frac{c}{2m\omega}$.

In such a way it brings to relation which is known in structure dynamic literature as transmissibility [39] and used for vibration isolation prediction. Plot for amplitude ratio in case of foundation base displacement excitation built in MATLAB for different damping ratio is shown in Fig. 2.

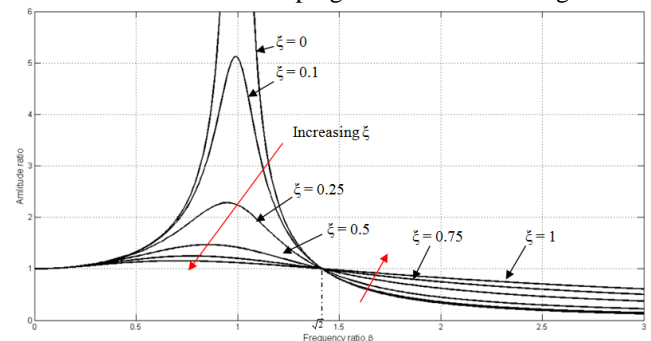


Fig. 2 Variation of amplitude of transmitted motion with frequency ratio and damping

It could be seen from the obtained numerical results that for frequency ratio equal to zero and till 0.5 the maximum amplitude of the single mass m will be equal or very close to amplitude of applied excitation. For undamped case the ratio tends to infinity at resonance, when $\beta=1$. However, the amplitude ratio of the system is less than unity for values of $\beta > \sqrt{2}$ for any amount of damping, while it is equal to unity in case of $\beta = \sqrt{2}$ for all damping levels. To sum up, when frequency ratio is $\beta < \sqrt{2}$, smaller damping ratios lead to larger values of systems maximum amplitude ratio. On the other hand, for $\beta > \sqrt{2}$, smaller values of damping lead to smaller values systems maximum amplitude ratio.

3 EXPERIMENTAL TEST RIG

To study the influence of foundation structure vibration acting on rotor structure, a special test rig with flexible suspension system was designed and built in laboratory of School of Energy and Power Engineering in Beihang University, Fig. 3.

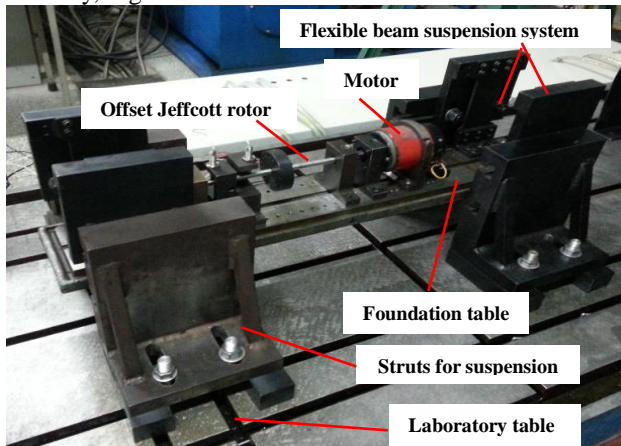


Fig. 3 Experimental test rig - general view: rotor mounted on flexible foundation (Experimental set 2)

In this test rig, simple single disk rotor on two cylindrical sleeve bearings ($L/D=1.5$), which are cooled by oil input from the hole at the top, with rigid supports (offset Jeffcott rotor) including driving motor was mounted on the plate type foundation table. Flexible beams were used to hang the table as a part of flexible suspension system, which were connected with rigid struts mounted on inertial metal-concrete laboratory table. Beams are rectangular plates 150 mm x 60 mm x 4 mm made of steel. Foundation table approximate weight was about 30 kg. To simplify the test set and get more clear experimental results the test rig was designed to move (be flexible) only in horizontal direction.

To understand the influence of foundation behavior on rotordynamics two configurations were observed: rotor mounted on rigid foundation (Fig. 4) and abovementioned rotor mounted on flexible foundation.

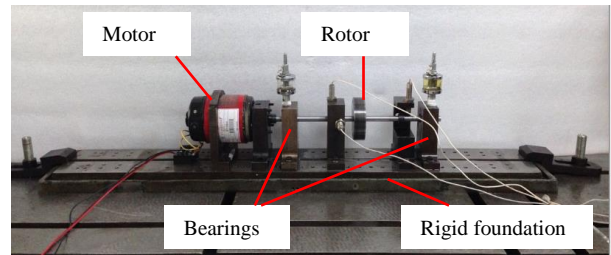


Fig. 4 Rotor mounted on rigid foundation (Experimental set 1)

To investigate parameters for the rotor with rigid foundation system configuration, flexible suspension system was unmounted and foundation table was rigidly fixed on inertial laboratory table. Due to keen interest to study behavior of the rotor system on different types of foundation structure measurement were made both for the shaft and disk in vertical and horizontal direction (absolute displacement), but due to limitation of the space only results for disk orbits are presented and compared in the current paper.

To understand how the foundation base movement influences on dynamics of the rotor system, rotor-foundation system from experimental set 2 was connected with centrally mounted shaker, Fig. 5. To simulate foundation displacement excitation sine wave signal was used for shaker. Probes to measure absolute displacement of the disk were located on laboratory table.

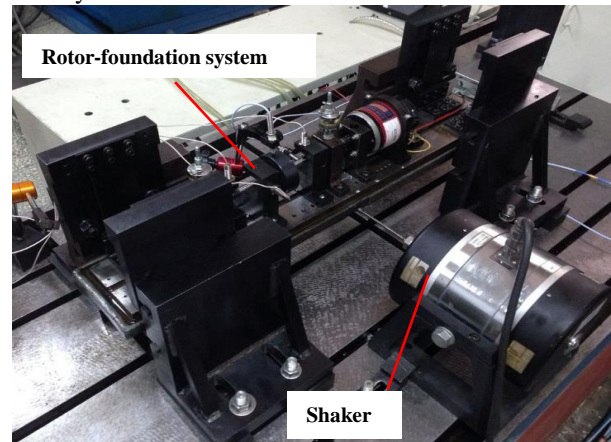


Fig. 5 Rotor-bearing-foundation system subjected to kinematic excitation (Experimental set 3)

General view on main elements of data acquisitions system, including probes location, is shown in Fig. 6.

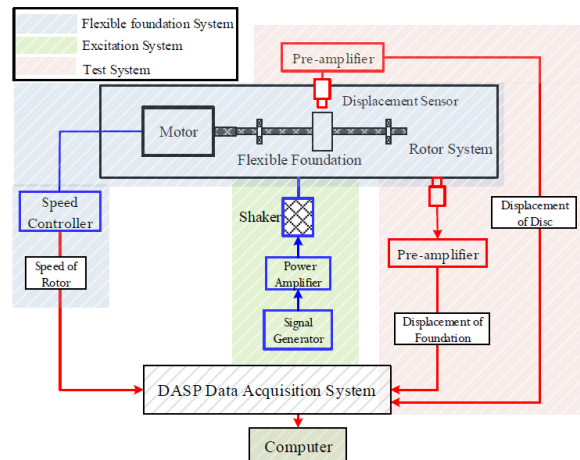


Fig. 6 Scheme of the data acquisition system

Changing amperage and frequency for the shaker, foundation structure was excited in horizontal direction, in order to investigate how the foundation structure kinematic excitation influence on rotor disk movement.

4. NUMERICAL SIMULATION

Creation of adequate mathematic model is an important stage for every research since accurate model can help significantly in prediction of dynamic behavior for real machine. Rotor model on rigid foundation was modeled in Dyrobes rotordynamic software. Rotor model consist from shaft and offset disk which was modeled as concentrated mass. Speed ring with a notch was used on test model for speed and phase measurements. General view on the model is shown in Fig. 7. Support stiffness was estimated based on static structural analysis for simplified support model build in ANSYS. Then for the rotor model linear isotropic supports with stiffness equal to $2E7$ N/m were used to define modal parameters of the system. Geometric properties for the main components of the model are summarized in form of Table 1.

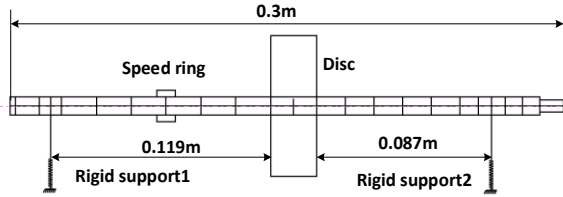


Fig. 7 Rotor model on rigid foundation

The shaft was modeled using beam elements, where the d.o.f.s are defined by $\{x\} = (u_x, u_y, \theta_x, \theta_y)^T$, hence each element was modeled by 8 d.o.f.s with two lateral translations (u_x, u_y) and two bending rotations (θ_x, θ_y).

Table 1 Geometric properties for rotor model

No	Name	Notation	Value
Shaft			
1	Total length	L	0.300 m
2	Midspan length	L_m	0.231 m
2	Diameter	D_s	0.010 m
3	Elastic modulus	E	$2 \cdot 10^5$ MPa
4	Density	ρ	8000 kg/m ³
Disk			
8	Mass	M_{disk}	0.792 kg
9	Outer diameter	D_{disk}	0.075 m
10	Axial length	L_d	0.02 m
11	Diametral inertia moment	I_d	$3,46841E-4$ kg*m ²
12	Polar inertia moment	I_p	$6.05539E-4$ kg*m ²

The bearing stiffness was set as support stiffness, hence bearing and damping matrices are:

$$[K]_{bear} = \begin{pmatrix} K_{xx} & K_{xy} & 0 & 0 \\ K_{yx} & K_{yy} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; [C]_{bear} = \begin{pmatrix} C_{xx} & C_{xy} & 0 & 0 \\ C_{yx} & C_{yy} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \quad (7)$$

Undamped critical speeds and mode shapes for rotor on rigid foundation for the first two ordinal modes are summarized in form of Table 2. Numerical results had shown that first critical speed is about 103 Hz which is lower, than rotor operation speed (150 Hz), in such a way rotor could be considered as flexible. Since the disk is offset from the middle of the shaft, both of the two mode shapes are affected by the disk gyroscopic moments.

Table 2 Predicted undamped critical speeds and mode shapes for model on rigid foundation

No	Mode description	Natural frequency, Hz	Critical speed, Hz	Critical speed, rpm	Mode shapes
1	Rotor first bending mode	103,47	103,6	6 216	
2	Rotor second bending mode	574,91	1532,4	91 945	

To verify the differences of natural frequency and critical speed, the Campbell diagram of the rotor was built, shown in Fig. 8. Taking into account disk offset position from the center of the shaft between the supports, the translation and rotation motion of the rotor are coupled and hence gyroscopic effect exist in two modes. However, for the first mode it's not so evident because offset of the disk from the center is not so big. At the same time it is obvious that natural frequencies associated with forward whirl increase as the rotor speed increase, while the natural frequencies associated with backward whirl decrease as the speed increase.

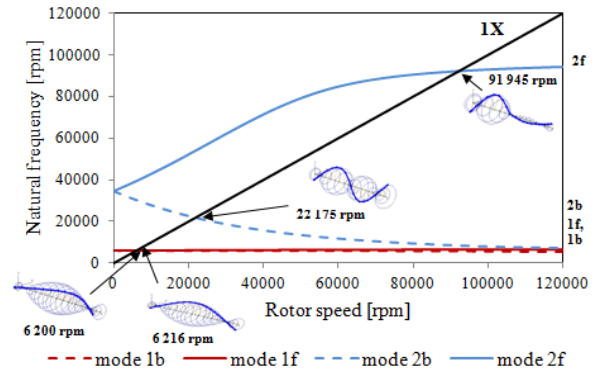
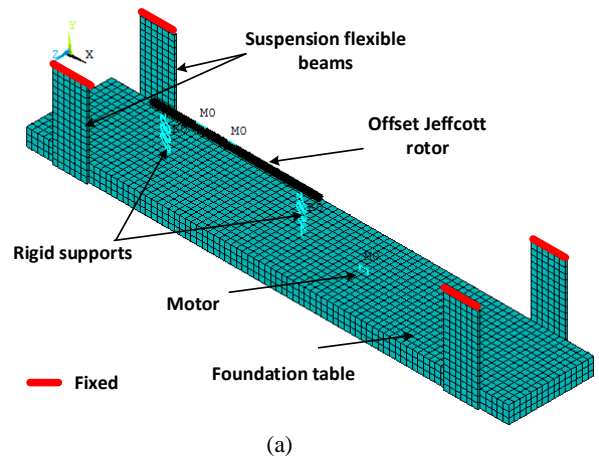


Fig. 8 Campbell diagram of rotor: b-backward; f-forward

To predict mode shapes and behavior of rotor mounted on flexible foundation two FEM models were created: model with plate type solid foundation and model with lumped mass foundation in ANSYS (Fig. 9– a) and Dyrobes (Fig. 9– b) respectively.



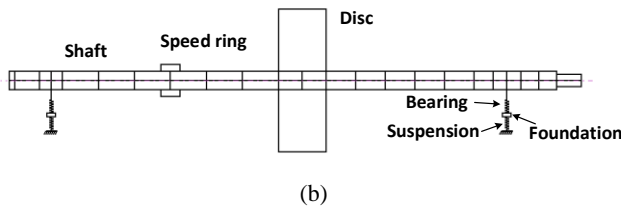


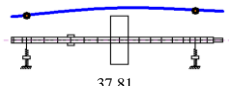
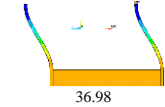
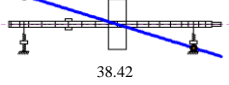
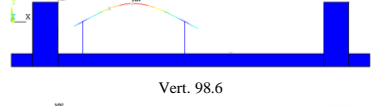
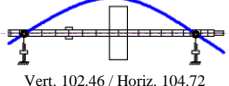
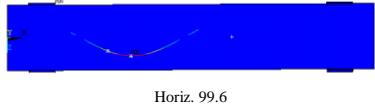
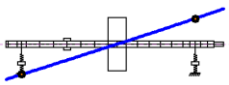
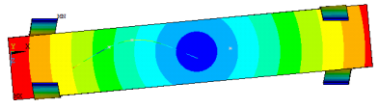
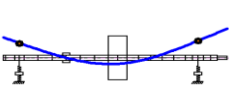
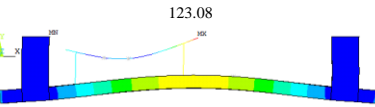
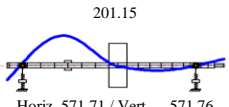
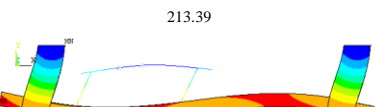
Fig. 9 FEM models for flexible foundation: a) Plate type solid foundation model; b) Lumped mass foundation model

For the first model foundation table was built in ANSYS using Brick186 elements, while for the rotor Beam188 elements were used. Mass21 with corresponding inertial moments and masses were used to model disk and speed ring. The rotor was connected with the table using Combine214 element which stiffness and damping matrixes are almost the same as (7). Location of the nodes corresponds with location of the supports on real test set. Because of significant weight, comparing with rotor, motor mass (almost 2.2 kg) was also included in the model and attached in corresponding node. Fixed supports were used on the end of flexible suspension beams to simulate the boundary condition.

For the second model the same rotor was used, but bearing nodes were connected with lumped masses which were used to model foundation. In such a way foundation structure mass, stiffness and damping matrix could be written as:

$$[M]_{found} = \begin{pmatrix} M_{xx} & 0 \\ 0 & M_{yy} \end{pmatrix}; [K]_{found} = \begin{pmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{pmatrix}; [C]_{found} = \begin{pmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{pmatrix}; \quad (8)$$

Table 3 Predicted natural frequencies and mode shapes for rotor-foundation structure

Lumped mass foundation model:			Plate type foundation model: $K_{sup}=2E7$ N/m		
Nº	$K_{sup}=2E7$ N/m, $K_{xx\ found}=0.905$ N/m, $K_{yy\ found}=2.45E1$ N/m	Mode shapes / Natural frequency, Hz	Mode description	Mode shapes / Natural frequency, Hz	
1	Foundation rigid cylindrical horizontal	 37.81	Foundation rigid cylindrical horizontal	 36.98	
2	Foundation rigid pivotal horizontal	 38.42	Rotor first bending	 Vert. 98.6	
3	Rotor first bending	 Vert. 102.46 / Horiz. 104.72	Foundation rigid pivotal horizontal	 Horiz. 99.6	
4	Foundation rigid pivotal vertical	 199.97	Foundation bending mode vertical	 123.08	
5	Foundation bending mode vertical	 201.15	Foundation axial mode	 213.39	
6	Rotor second bending	 Horiz. 571.71 / Vert. 571.76		 295.81	

5 EXPERIMENTAL MEASUREMENTS AND RESULTS

5.1 Numerical model validation

Run up/ run down measurements were made for experimental set 1 and 2 in the speed range 0-200 Hz to define the difference between rigid and flexible foundation types. Comparison of experimental and numerical results

Combining (7)-(8) equation of motion for rotor-foundation system could be obtained as:

$$\begin{pmatrix} M_{rr} & 0 \\ 0 & M_{ff} \end{pmatrix} \begin{Bmatrix} \ddot{x}_r \\ \ddot{x}_f \end{Bmatrix} + \begin{pmatrix} C_{rr} & C_{rf} \\ C_{fr} & C_{ff} \end{pmatrix} \begin{Bmatrix} \dot{x}_r \\ \dot{x}_f \end{Bmatrix} + \begin{pmatrix} K_{rr} & K_{rf} \\ K_{fr} & K_{ff} \end{pmatrix} \begin{Bmatrix} x_r \\ x_f \end{Bmatrix} = \begin{Bmatrix} F_R \\ F_F \end{Bmatrix} \quad (9)$$

, where M_{rr} , C_{rr} , K_{rr} - are the mass, damping and stiffness matrix of the rotor; M_{ff} , C_{ff} , K_{ff} - are the mass, damping and stiffness matrix of the foundation; C_{rf} , C_{fr} , K_{rf} , K_{fr} - are cross-coupled damping and stiffness matrix of rotor and foundation interaction; x_r - is the d.o.f. vector for the rotor structure displacements, while x_f - is the vector of foundation physical coordinates; F_R , F_F - are the vectors of external forces acting on rotor and foundation respectively.

Natural frequencies and approximate mode shapes for the first 6 ordinal modes are summarized in form of Table 3. Obtained results had shown that for both models rotor first flexural mode natural frequency is about (100 – 104 Hz) while first foundation mode is near 37 Hz. At the same time there is some mismatch in modes. Lumped mass foundation model predicts appearance of two foundation pivotal modes (which correspond to resonances of springs in vertical and horizontal directions) while for solid foundation model there is only one in horizontal plane.

numerical model. One can also notice that there is difference for the critical speed for run up and run down measurements, caused by the “lag phenomenon” which is common for the rotor systems [27].

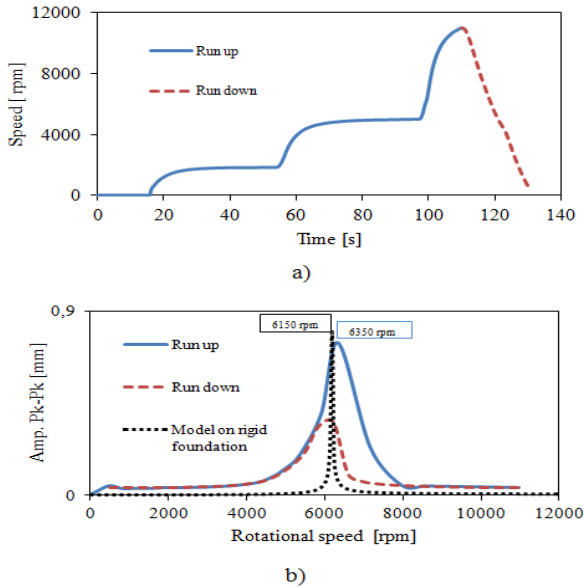


Fig. 10 Run up/run down characteristics for the rotor on rigid foundation: a) Run up/ run down speed vs. time b) Amplitude measured on the disk in vertical direction

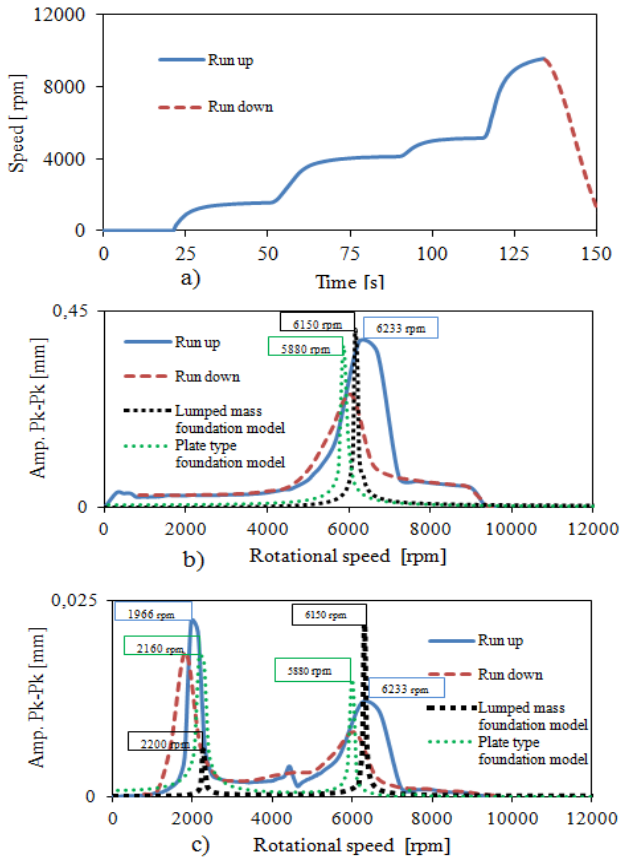


Fig. 11 Run up/run down characteristics for the rotor on rigid foundation: a) Run up/ run down speed vs. time b) Absolute amplitude measured on the disk in vertical direction c) Absolute amplitude measured on foundation in horizontal direction

Placing the rotor on flexible foundation brings to decrease of the amplitude on the critical speed, at the same time foundation modes could appeared in the operating range of the machine. Rotor orbits measured on the disk for rigid foundation structure are shown in Fig. 12–a. Four different speeds were chosen for analysis: two sub-critical speeds (23Hz and 75Hz) – lower and higher than foundation resonance, one near rotor critical speed (95Hz), and one supercritical speed which corresponds to rotor operational speed.

5.2 Development of numerical model for foundation kinematic excitation

To predict what would be the motion of the rotor disk if foundation structure is subjected to kinematic excitation simplified numerical model for single disk Jeffcott rotor on massless shaft, Fig. 13-a, could be used:

$$\begin{cases} M\ddot{x} + C_x\dot{x} + K_{eqv}x = me\Omega^2 \cdot \cos(\Omega t) \\ M\ddot{y} + C_y\dot{y} + K_{eqv}y = me\Omega^2 \cdot \sin(\Omega t) \end{cases} \quad (10)$$

, where M is disk mass, $K_{eqv} = \frac{K_{shaft}}{1 + K_{shaft} / 2K_{bear}}$ –

equivalent stiffness, K_{shaft} – shaft stiffness, K_{bear} – bearing stiffness, C_x, C_y – damping coefficients for bearing, me – disk unbalance, Ω – rotational speed. The benefit of this approach is that both equations are decoupled and could be solved separately. Using $M = 0.985$ kg as mass of the disk, isotropic bearings with $K_{eqv} = 4.05 \text{ E5 N/m}$, the first critical speed for the model could be estimated using equation (11). Results for rotor on rigid bearings and bearings with K_{eqv} are summarized in Table 4.

$$\omega_{cr} = \sqrt{\frac{K_{shaft}}{m}} \frac{1}{\sqrt{1 + K_{shaft} / 2K_{bearings}}} \quad (11)$$

Table 4 Critical speeds for the Jeffcott rotor model

Critical speed on rigid bearings		Critical speed for K_{eqv}	
rad/s	rpm	rad/s	rpm
644,51	6154,61	641,24	6123,4

Above mentioned model was modified based on proposed in section 2 theory about foundation structure kinematic excitation, Fig. 13-b. In such a way mathematical model for the system subjected to foundation displacement excitation in horizontal plane could be written as:

$$\begin{cases} M\ddot{x} + C_x\dot{x} + K_{eqv}x - K_{eqv}u_f - C_x\dot{u}_f = me\Omega^2 \cdot \cos(\Omega t) \\ M\ddot{y} + C_y\dot{y} + K_{eqv}y = me\Omega^2 \cdot \sin(\Omega t) \end{cases} \quad (12)$$

, where u_f – is displacement excitation of the foundation structure. At the same time this system could be written in short form as:

$$[M]\ddot{q} + [C]\dot{q} + [K]q = \{F_{unb}\} + \{F_{found}\} \quad (13)$$

, where $[M]$, $[C]$, $[K]$ – are the mass, damping and stiffness matrixes, $\{q\} = (x, y)^T$ – d.o.f vector in general coordinates, $\{F_{unb}\}$ – vector of residual unbalance, $\{F_{found}\}$ – vector of foundation displacement excitation.

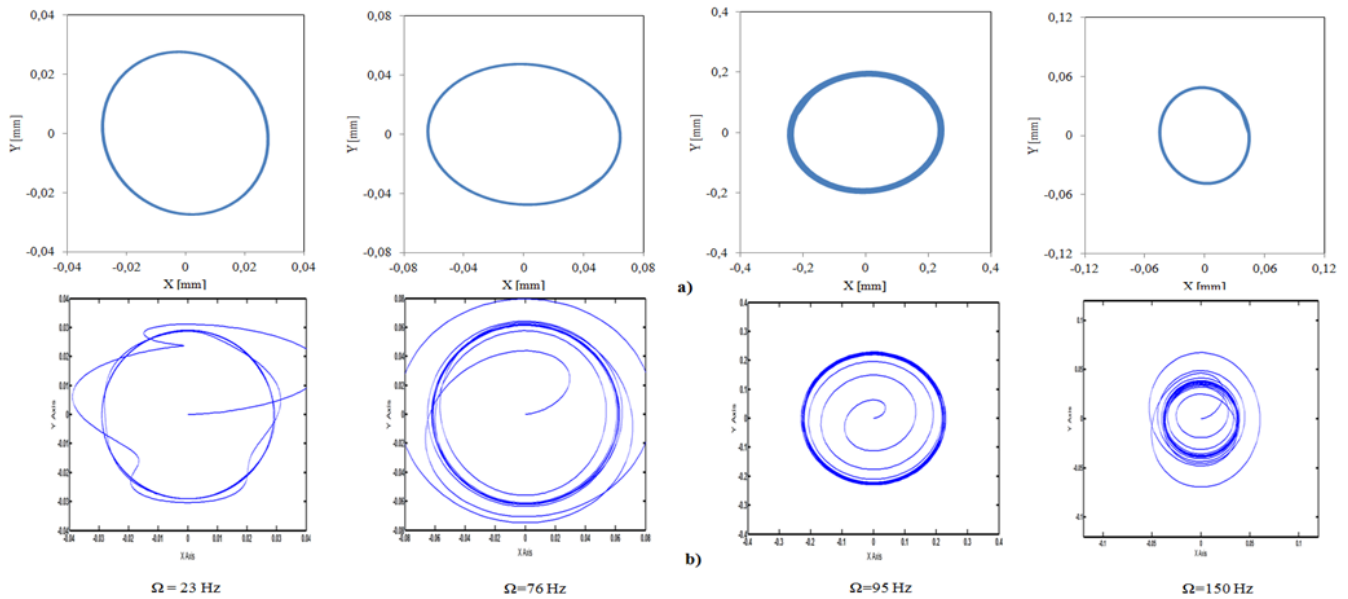


Fig. 12 Rotor absolute displacement orbits for disk: a) measured on rigid foundation b) numerical model for disk orbit - with initial unbalance on rigid foundation, $t=10$ s; Ω - rotor speed

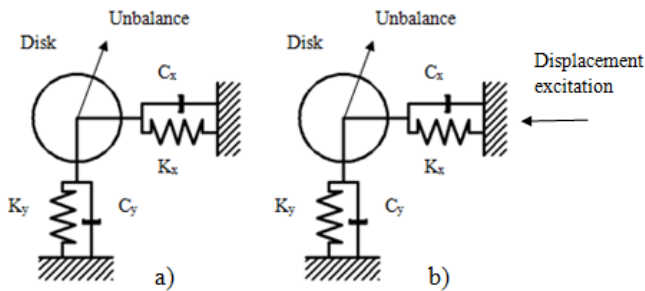


Fig. 13 Simplified rotor model: a) Rotor model subjected to unbalance; b) Rotor model subjected to unbalance and foundation excitation in horizontal direction

Abovementioned model was built in MATLAB Simulink to observe behavior of rotor's disk displacement orbit. Results of numerical validation for the case when rotor rotates with initial unbalance $1.5 \cdot 10^{-2}$ kg-mm are shown in Fig. 12. Comparison with previously obtained results for rotor on rigid foundation (Fig. 12-a) shows that general size of the orbits in vertical and horizontal direction is almost the same (Fig. 12-b).

Differences in shape and exact value of maximum displacement could be explained because of cylindrical oil bearings which were used in the test rig and hence bearing stiffness is not the same in vertical and horizontal direction, while to simplify the calculations linear isotropic bearings were used for modeling. Since orbits obtained numerically for different speeds were quite similar with orbits for the disk from experiment, it was decided to use this model with the same parameters of stiffness and damping for the simulation of foundation excitation.

Spectrograms of rotor vibration, both for experiment and simulation, are shown in Fig. 14. Comparison with numerical model also showed good agreement for all four rotor speeds.

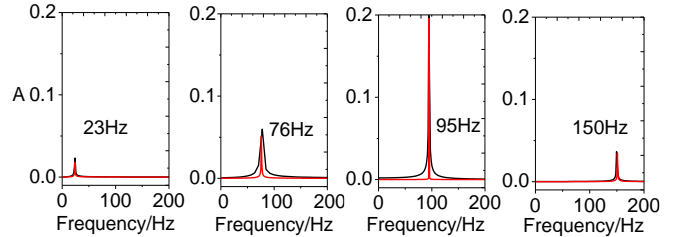


Fig. 14 Frequency domain analysis for rotor on rigid foundation (black line - experiment, red line - the simulation)

Foundation displacement excitation was made in MATLAB using sine wave signal in accordance with theory of displacement excitation which was described earlier in section 2. Displacement excitation was set to be applied not in phase with unbalance excitation. Ode4 (Runge-Kutta) solver with time step $1 \cdot 10^{-5}$ s was chosen for analysis.

Results of predicted disk absolute orbit for the rotor rotated on constant speed with constant foundation displacement are shown in Fig. 15.

Applied foundation excitation showed that for 1 sec rotor will make 23 ($\Omega=23$ Hz) and 150 ($\Omega=150$ Hz) rotations respectively with complicated trajectory due to rotor jumps. Obtained results revealed that foundation displacement defines disk's maximum deviation in the plane of applied excitation while increase of foundation frequency excitation brings to increase of rotor jumps when he alters trajectory of movement due to applied excitation and influence of initial unbalance.

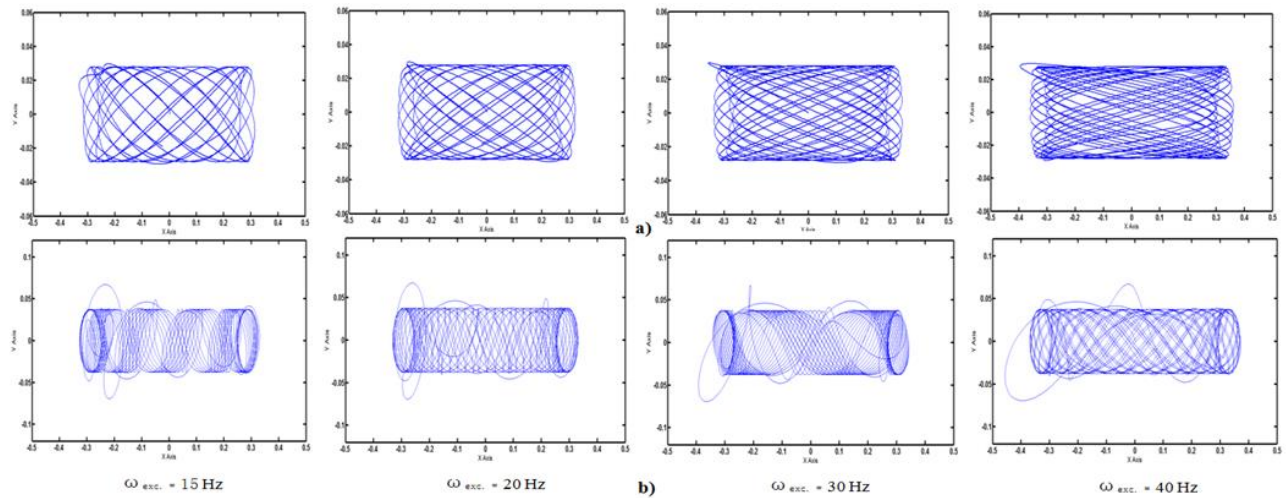


Fig. 15 Rotor disk absolute displacement orbits obtained numerically when rotor was subjected both to initial unbalance and foundation displacement excitation, $A_{\text{found}}=0.2\text{mm}$: a) $\Omega=23\text{Hz}=\text{const}$, $t=1\text{s}$; b) $\Omega=150\text{ Hz}=\text{const}$, $t=0.5\text{ s}$; Ω - rotor speed; ω_{exc} – frequency of foundation excitation

5.3. Experimental measurements

In order to check results of the numerical model measurements were made on experimental test set 3. Foundation displacement excitation was performed for two cases with constant displacement for foundation and constant rotor speed (case 1 - subcritical operation, $\Omega=23\text{Hz}$; case 2- supercritical operation, $\Omega=150\text{Hz}$), while frequency of excitation was changing.

Obtained results (Fig. 16) have shown that for both cases disk absolute displacement orbits in general look quite similar with those which were predicted by numerical model (Fig. 15). At the same time the trajectory for both cases has a small

slope in relation to plane of excitation, while for numerical model the disk was moving only in plane where displacement excitation was applied. Probable cause for that is foundation small movement in vertical direction. Although it was assumed that table moves only in horizontal plane, flexibility of system and vertical alignment error with shaker makes possible vertical small deviation of table during the test. Nevertheless displacements in vertical direction are small enough in comparison with horizontal and hence could be neglected.

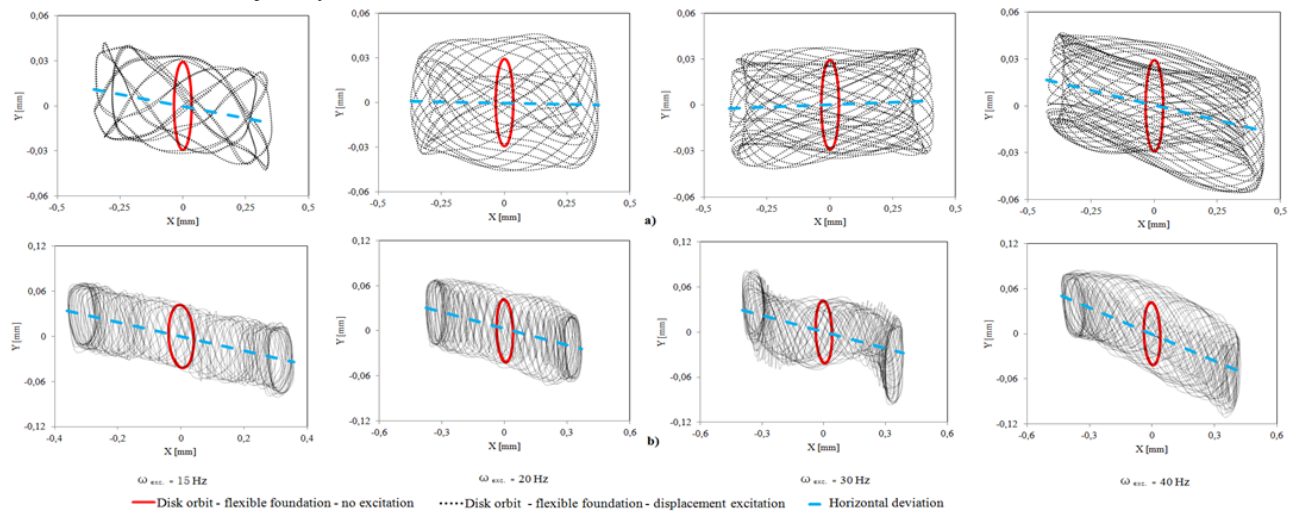


Fig. 16 Experimental results: influence of frequency of foundation movement on rotor disk absolute displacement, $A_{\text{found}}=\text{const}=0.2\text{ mm}$, $t=1\text{s}$; a) $\Omega = 23\text{Hz}$ b) $\Omega= 150\text{Hz}$; Ω - rotor speed; ω_{exc} – frequency of foundation excitation

Experimental results confirmed the main conclusions which were made for numerical model, and predicted maximum displacement was almost the same. In addition, comparison of numerical and experimental results for several rotor rotations on slow speed showed quite good agreement, Fig. 17, since the general shape of the disk motion coincides almost for all cases of excitation.

In order to understand frequency content of vibration signal, measured on the disk for the rotor speed $\Omega=23\text{Hz}$, the frequency spectrograms both for experimental and numerical results of the rotor in the plane of applied excitation (horizontal direction) were built, Fig. 18.

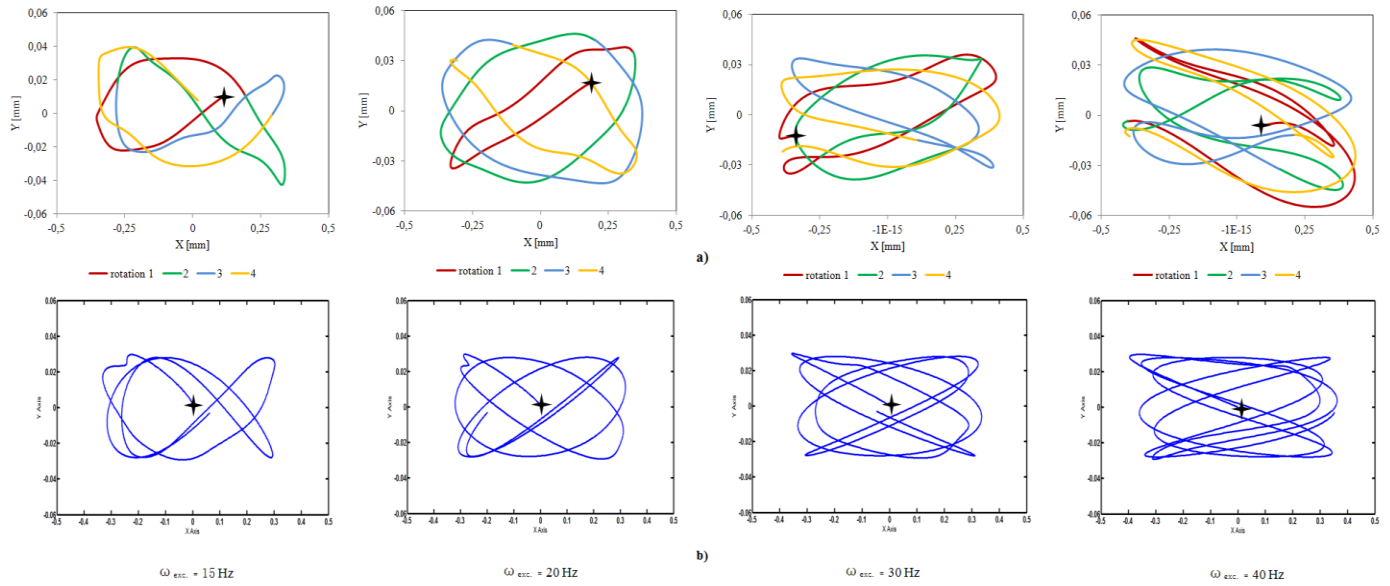


Fig. 17 Comparison of disk trajectory movement obtained for several rotations, $A_{\text{found}}=0.2$ mm, $\Omega=23$ Hz, $t=0.168$ s, when rotor-foundation system was subjected to displacement excitation: a) experimental results for disk absolute displacement; b) numerical results (asterisk – start point)

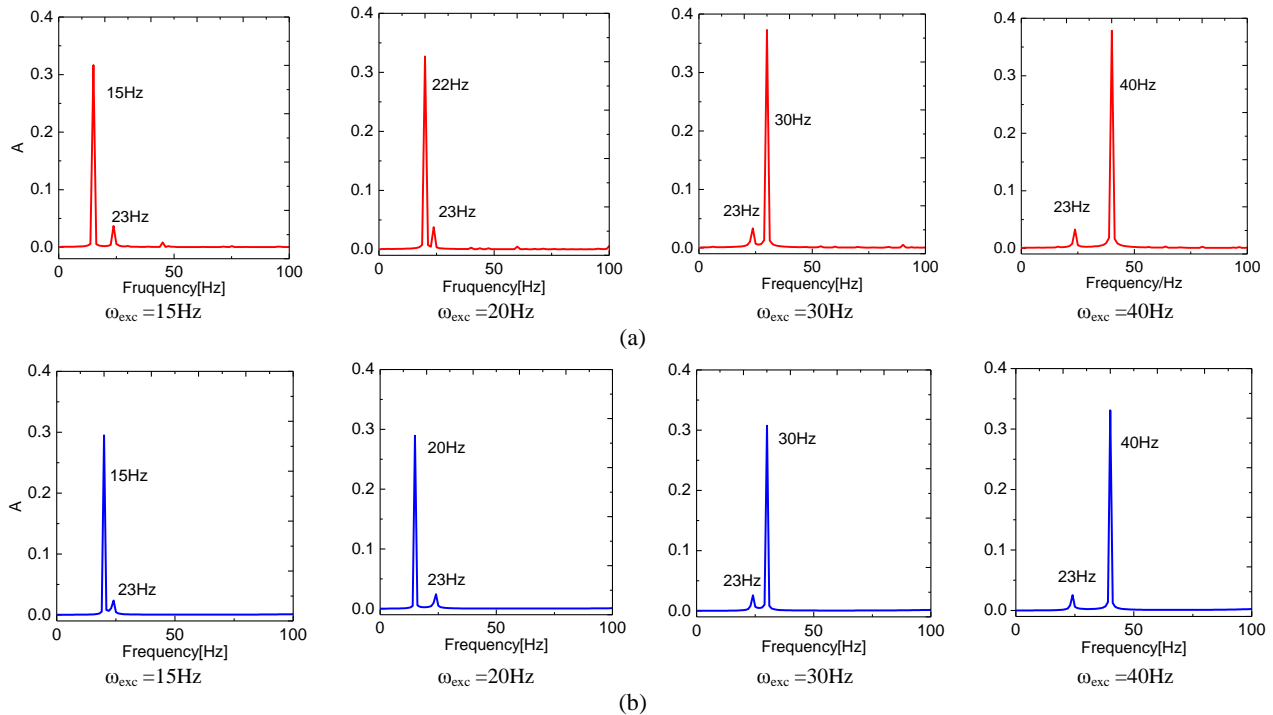


Fig. 18 Frequency domain analysis for rotor disk vibration signal when rotor-foundation system was subjected to displacement excitation, $\Omega=23$ Hz: a) experimental; b) numerical results; Ω - rotor speed; ω_{exc} – frequency of foundation excitation

Obtained results have shown that in both cases (experiment and simulation results) there are only two main frequencies in the vibration signal. One is inherent to residual rotor unbalance (peak equal to rotor speed) and another one corresponds to foundation excitation (peak with frequency of applied excitation). Existence of the last one in vibration

signal measured on the disk proves that there is interaction between foundation and rotor structure.

In the same manner influence of foundation displacement amplitude was investigated in more detail. Experimental measurements were also made for subcritical and supercritical rotor operation, Fig. 19.

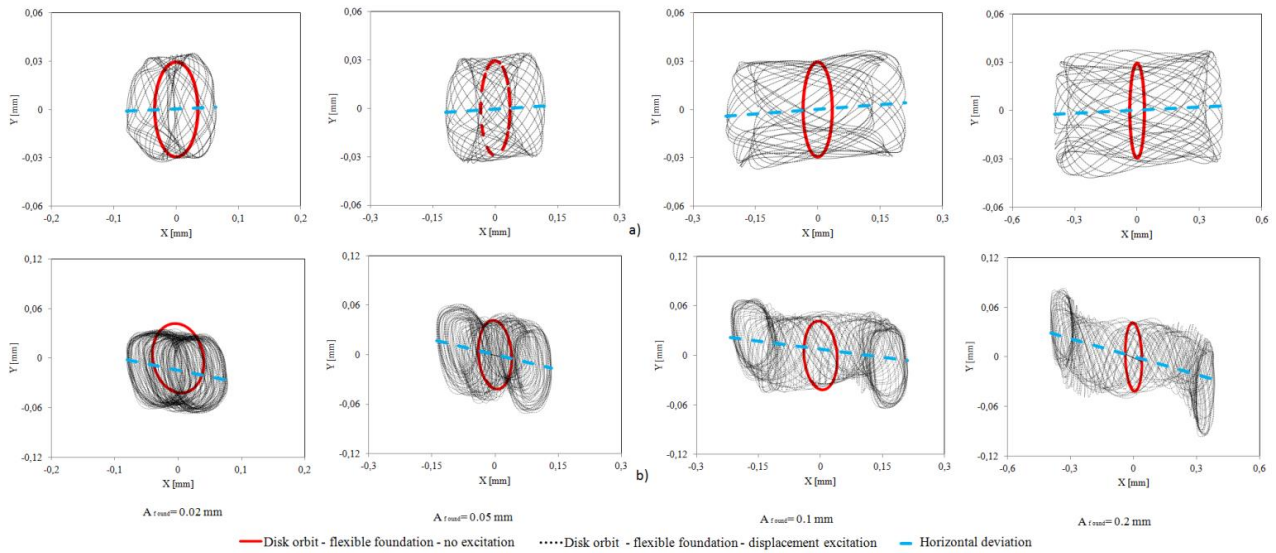
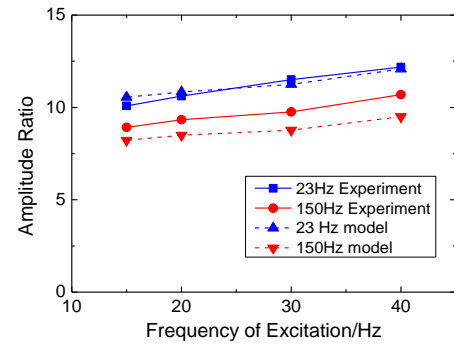


Fig. 19 Experimental results: influence of foundation displacement on absolute movement of rotor disk, $\omega_{exc} = \text{const} = 30\text{Hz}$, $t = 1\text{s}$: a) $\Omega = 23\text{Hz}$; b) $\Omega = 150\text{Hz}$; Ω - rotor speed; ω_{exc} - frequency of foundation excitation; A_{found} - amplitude of foundation displacement

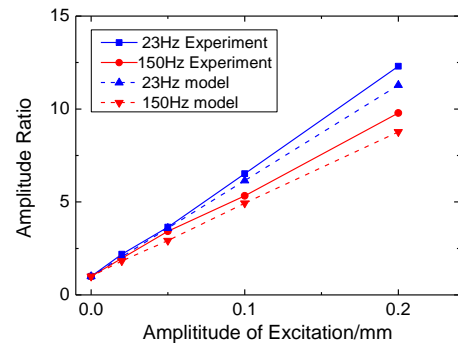
Obtained experimental results have proved that influence of amplitude of foundation vibration is quite significant, since it defines approximate size of maximum rotor deviation. At the same time for slow rotor rotation disk orbit is a complicated curve with rapid turns as a result of foundation excitation, while for high speed rotation disk moves on trajectory close to helical in the plane of applied excitation, what was also predicted by numerical model, Fig. 15-b. It is also clear that when ω_{exc} is higher than rotor rotation speed the orbit shape transform to complicated motion which could rapidly turns the direction. In such a way displacement excitation with high frequency is more dangerous for slow rotor rotation.

To understand how foundation displacement excitation influence on rotor disk motion more evident, maximum displacement of the disk for each case were also evaluated, Fig. 20. The plot present comparison of maximum amplitude of disk displacement when the rotor was subjected to displacement excitation to amplitude of the disk when it was on rigid foundation without excitation. Comparison of amplitude in this way is important since although some other parts such as case or machine body parts are located on the same foundation and hence also move with it when foundation structure is subjected to kinematic excitation, but rotor amplitude can change significantly and exceed allowable clearance, what in turn can lead to damage of important parts such as blades, seals, rotor journals etc.

It could be seen from the obtained results that increase of frequency of foundation excitation can bring to increase of rotor absolute amplitude up to 12 times. Comparison with numerical model showed good agreement for rotor low speed, (Fig. 20-a), but at the high speed model showed much lower values of amplitude. In the meantime influence of amplitude of foundation excitation is also significant, Fig. 20-b.



a)



b)

Fig. 20 Absolute amplitude of the disk: a) influence of foundation frequency b) influence of foundation amplitude

Increase of foundation displacement amplitude can alter amplitude of the disk more than in 10 times. Results obtained by the model were lower in comparison with experiment but they reflect the general behavior of the rotor-foundation system quite good. Moreover, despite of significant difference between rotor rotational speeds amplitude ratio for both cases ($\Omega = 23\text{ Hz}$ and $\Omega = 150\text{ Hz}$) increases almost linear.

CONCLUSION

The foundation kinematic excitation can significantly alter behavior of the rotating machine and in such a way further work about creation of adequate mathematic modeling is important. Models can help to estimate approximate influence of excitation parameters what in turn can help to make changes in the construction (e.g. acceptable clearance for seals, blades, etc.) on the stage of design in order to reduce the risk of machine damage if excitation is unavoidable and receive more safe and reliable operation of the machine and its components under normal conditions. To sum up:

- Based on the obtained numerical and experimental results one can conclude that foundation displacement amplitude defines rotor maximum deviation in the plane of applied excitation;
- Frequency of foundation displacement excitation has significant influence on the shape of rotor absolute displacement trajectory. When rotor speed is in the same order of magnitude as excitation frequency, increase of later will bring to increase of rotor jumps and rotor trajectory will have rapid turns. In turn, when the speed of rotor is significantly higher than excitation frequency, rotor orbit will have helical shape which moves in plane of applied excitation;
- Proposed numerical model developed in MATLAB showed good agreement in reflecting of general behavior for disk deviation in comparison with experiment results obtained for rotor low and high operational speed. Nevertheless, tuning of the model is still necessary, since the model didn't predict exact value of maximum disk deviation amplitude when the foundation base was subjected to displacement excitation;

Finally it could be said that foundation excitation is dangerous because it can lead to rotor structure unpredicted motion when perfectly balanced rotor will deviate from its steady motion what it turn can lead to its damage. Obtained numerical results for rotor-foundation model showed good agreement in prediction of general behavior of rotor motion when the system was subjected to displacement excitation. Hence proposed in the current paper methodology for rotor-foundation system modeling could be further used for creation of more complicated models, e.g. for real aero engines.

ACKNOWLEDGEMENTS

This work is financially supported by the National Natural Science Foundation of China, the grant No. is 51475021.

REFERENCES

[1] Lund J.W. The stability of an elastic rotor journal bearings with flexible, damped supports. *Journal of Applied Mechanics*, Vol. 32, Transaction ASME, Series E 87, 1965: 911-920.

[2] Gunter E.J. Lund's Contribution to rotor stability: The indispensable and Fundamental Basis of Modern Compressor Design. *Journal of Vibration and Acoustics*, ASME, 2003, 10: 462-470.

[3] Gunter E.J. The influence of internal friction on the stability of high speed rotor. *Journal of Engineering for Industry*. Transactions of ASME, Series B 89, 1967: 683-688.

[4] Armentrout R.W., Gunter E.J. Transient modal analysis of nonlinear rotor-bearing system. Proceedings of the 17th International Modal Analysis Conference, February 8-11, 1999, Kissimmee, Florida.

[5] Chen W.J. Energy analysis to the design of rotor-bearing system. Proceedings of International Gas Turbine and Aeroengine Congress & Exposition, June 5-8, 1995, Houston, Texas.

[6] Nicholas J.C., Barrett L.E., Whalen J.K. Improving critical speed calculations using flexible bearing support FRF compliance data. 15th Turbomachinery symposium, Texas A&M, 1986: 69-78.

[7] Nicholas J.C. Utilizing dynamic support stiffness for improved rotordynamic calculations. Proceedings of the 17th International Modal Analysis Conference: 256-262.

[8] Kang Y.T., Chang Y.-P., Tsai J.-W., Mu L.-H., Chang Y.-F. An investigation in stiffness effects on dynamics of rotor-bearing-foundation systems. *Journal of Sound and Vibration*, 2000, 231(2): 343-374.

[9] Qingkai Han, Xiao Dong, Bangchun Wen Resonance capture of rotor system mounted on an elastically supported base. Proceedings of the 8th IFToMM International Conference on Rotor Dynamics, September 12-15, Seoul, Korea, 2010: 904-910.

[10] Vasquez J.A., Barrett L.E., Flack R.D. Including the effect of flexible bearing supports in rotating machinery. *International Journal of Rotating Machinery*, 2001, vol.7: 223-236.

[11] Vasquez J.A., Barrett L.E. Transfer function representation of flexible supports and casing of rotating machinery. Proceedings of 17th International Modal Analysis Conference, 8-11, February, 1999, Kissimmee, Florida.

[12] Lees A.W., Friswell M.I. The evaluation of rotor imbalance in flexible mounted machines. *Journal of Sound and Vibration*, 1997, 208(5): 671-683.

[13] Edwards S., Lees A.W., Friswell M.I. Experimental identification of excitation and support parameters of a flexible rotor-bearings-foundation system from a single run-down. *Journal of Sound and Vibration*, 2000, 232(5): 963 – 992.

[14] Lees A.W., Price E.D. Identification of rotor dynamic machinery – a laboratory trial. 23rd IMAC conference, Orlando, Florida, USA, February 2005.

[15] Cavalca K.L., Cavalcante P.F., Okabe E.P. An investigation on the influence of the supporting structure on the dynamics of the rotor system. *Journal of Mechanical Systems and Signal Processing*, Elsevier, 2005, 19: 157-174.

[16] Cavalcante P.F., Cavalca K.L. A method to analyze the interaction between rotor-foundation systems. IMAC98, 16th International Modal Analysis Conference, Santa Barbara, USA, February 1998: 775-781.

[17] Cavalca K.L., Okabe E.P. On analysis of rotor-bearing-foundation system. IUTAM Symposium on Emerging Trends in Rotor Dynamic, 2011: 89-101.

[18] Feng N., Hahn E. J., Sestieri A. A combined finite element/transfer matrix approach for including foundation effects on the vibration behavior of rotating machinery. Proceedings of Sixth International Conference on Vibrations in Rotating Machinery, Institution of Mechanical Engineers, 1992, 529-534.

[19] Feng N. S., Hahn E. J. Including foundation effect on the vibration behavior of rotating machinery. *Mechanical System Processing*, 9(3), 1995: 243-256.

[20] Feng N. S., Hahn E.J. Experimental identification of the foundation dynamic stiffness parameters of rotor-bearing-foundation systems. Proceedings of the 6th International symposium on transport

- phenomenon and dynamics of rotating machinery, Honolulu, 1996: 48-57.
- [21] Feng N. S., Hahn E.J. Turbomachinery foundation identification using foundation modal parameters. Proceedings ISMA-21 Noise and Vibration Engineering Conference, 1996, 1503-1513.
- [22] Pennacchi P., Bachschmid N., Vania A., Zanetta G.A., Gregori L. Use of modal representation for the supporting structure in model-based fault identification of large rotating machinery: part 1 – theoretical remarks. *Mechanical Systems and Signal Processing*, 20, 2006: 662-681.
- [23] Pennacchi P., Bachschmid N., Vania A., Zanetta G.A., Gregori L. Use of modal representation for the supporting structure in model-based fault identification of large rotating machinery: part 2 – application to a real machine. *Mechanical Systems and Signal Processing*, 20, 2006: 682-701.
- [24] Gasch R. Vibration of large turbo-rotors in fluid-film bearings on elastic foundations. *Journal of Sound and Vibration*, 1976, 47(1), 53-73.
- [25] Stephenson R.W., Rouch K.E., Generating matrices of the foundation structure of a rotor system from test data. *Journal of sound and vibration*, 8 (5), 1992: 467-484.
- [26] Murphy B.T., Vance J.M. An improved method for calculating critical speeds and rotordynamic stability of turbomachinery. *Journal of Engineering for Power*, 1983, 153(7): 591-595.
- [27] Kramer E. Dynamics of rotors and foundations. Springer-Verlag, Berlin, 1993.
- [28] Kostyk A.G. Shatochin V.F. Ivanov N.M. Prediction of dynamic stability level for heavy power machines, Proceedings of Moscow Power University, 127, 1972 (in Russian).
- [29] Kostyk A.G. Shatochin V.F. Vibration of turbomachinery on foundation from unbalance of rotor train. *Heat Power Engineering*, 12, 1971: 79-82 (in Russian).
- [30] Kostyk A.G., Shatochin V.F. Calculation of transient oscillation for rotor train subjected sudden unbalance. Proceedings of Moscow Power University, 99, 1972: 29-34 (in Russian).
- [31] Savinov O.A. Modern design of foundation structures for rotating machines and its computation. Leningrad, Stroiizdat, 1989 (in Russian).
- [32] Shulzhenko N.G., Vorobiev U.S. Numerical analysis of vibration for turbine units - foundation systems. Kiev: Naukova dumka.-1991 (in Russian).
- [33] Shulzhenko N.G., Vorobiev U.S. Study on vibration for element systems of turbine units, Kiev: Naukova dumka.-1978 (in Russian).
- [34] Ehehalt U., Lunenburg B., Staubach R., Daniel C., Stackeljan J., Woschke E. Methods to incorporate foundation elasticities in rotordynamic calculations. SIRM 2009 - 8th International Conference on Vibration s in Rotating Machines, 2009.
- [35] Kuemmler H., Siegl G., Woywode P. Influence of elastic foundation structures on the rotor dynamics of drive trains. Petroleum and Chemical Industry Conference Europe - Electrical and Instrumentation Applications, June 10-12, 2008.
- [36] Krüger, T. D., Liberatore, S., Knopf, E., Clark, A. Consideration of Complex Support Structure Dynamics in Rotordynamic Assessments. Proceedings of ASME Turbo Expo 2013: Turbine Technical Conference and Exposition, June, 2013.
- [37] Hong J., Shaposhnikov K.V., Zhang D., Ma Y. Theoretical modeling for a rotor-bearing foundation system and its dynamic characteristics analysis, Proceedings of 9th IFToMM ICORD 2014, September 22-25, Milan, Italy.
- [38] Clough R.W., Penzien J. Dynamics of structures, 3rd edition, Computers & Structures, Inc., 2003.
- [39] Humar J.L. Dynamics of Structure. Prentice Hall, Inc. 1990.
- [40] Shatochin V.F., Zimmerman S.D. Turbine rotor vibration due to unsteady kinematic excitation. Calculation method. *Aerospace technics and technology*, 8 (34), 2006: pp. 57-68 (in Russian).
- [41] Shatochin V.F., Zimmerman S.D. Development of disaster prevention system for power plant units. Part 1 Analysis of catastrophic accidents and formulation of the problem. *Aerospace technics and technology*, 10(26), 2005: pp. 19-31 (in Russian).
- [42] Chang-Sheng Zhu. Experimental investigation on dynamic behavior of active magnetic bearing-rotor system subject to base vibration. *Acta Aeronautica Et Astronautica Sinica*, 2004, 25(2): pp. 168-171(in Chinese).
- [43] Ke-Jian Jiang, Chang-Sheng Zhu, Xiao-Li Qiao, Liang-Liang Chen. Suppressing vibration due to base motion for active magnetic bearings-rotor system equipped on moving carrier. *Journal of Mechanical Engineering*, 2014, 50(11): pp. 108-118 (in Chinese).
- [44] Guang-Chi Ying, Guang Meng, Jianping Jin, Xin-Hua Long. Turbocharger rotor dynamics considering engines foundation excitation. *Journal of Vibration and Shock*, 2009, 28(4): pp. 18-24(in Chinese).
- [45] Guang-Chi Ying. Turbocharger's base excitation identification and rotor dynamics investigation. Ph. D. dissertation of Shanghai JiaoTong University, 2008 (in Chinese).
- [46] Pryadko S.V., Chvorost V.A. Dynamic of rotating machines subjected to seismic impact. *Herald of SumDU*, 1994: pp. 75-80.(in Russian)
- [47] Fang Ming, Wang Tao, Li Hui. Dynamic behavior of foundation considering full interaction among facility, structure and soil. Proceedings of 15th WCEE conference, Lisbon, 2012.
- [48] Weiming Liu, Nowak M. Dynamic behavior of turbine-generator-foundation systems. *Earthquake engineering and structural dynamics*, 24, 1995: pp. 339-360.
- [49] Srinivasan V., Soni A.H. Seismic analysis of a rotor-bearing system, *Earthquake Engineering & Structural Dynamics*, Volume 12, Issue 3, 1984, pp. 287-311.
- [50] Soni A.H., Srinivasan V. Seismic analysis of rotating mechanical system, Proceedings of the 8th world conference on earthquake engineering, San Francisco, California, 1984, pp. 1205-1215.