

## EXPERIMENTAL INVESTIGATION ON DYNAMICAL RESPONSE OF AN OVERHUNG ROTOR DUE TO SUDDEN UNBALANCE

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### ABSTRACT

Blade loss is a typical extreme load in the turbo machinery, which can cause intense vibration in rotor and huge loads in supporting system due to the sudden unbalance applied on the disk. An overhung rotor-support system is built to study the dynamic characteristics of rotor and support experimental system under the sudden unbalance excitation. The responses due to a mass loss are tested respectively both in subcritical state and in supercritical state, further the orbits of the rotor and load transmitting process on the stator are obtained and investigated. Moreover, the impact effect of response achieved by a mechanical model is presented to compare with the test result. The results show that sudden unbalance can induce an impact effect on the rotor system with response containing rotational speed frequency and natural frequency in frequency domain. The impact effect is more evident for flexible rotor than the rigid rotor, and the vibration response exhibits local effect. As a consequence the paper provides a reference and basis for the dynamical design and analysis for flexible overhung rotor system suffering sudden unbalance.

### INTRODUCTION

The release of fragments or even the whole blade from the rotor is a typical and severe failure mode in the rotating machinery which leads to a high level vibration of the whole system, and even many different types of damages. Blade loss which happens in aero engines is a major safety concern in commercial civil aviation. The authorities request the manufactories to run a successful fan blade loss test that can demonstrate the capability of withstand such severe loads for each type of engines. Nevertheless, the whole engine test is a costly and complex process, so a lot of pre-analyses, simulations should be done to guide the structural safety design. Thus the reliability of mechanical behaviors and characteristics also need the data measured in the simpler

mechanism tests to validate.

During the past decades, studies have been done on the topic of blade loss, including containment of casing[3], transient vibration of rotor system[4,5], coupling vibration of rotor with stator[6,7] and survive ability of the rotor with huge unbalance[8] etc. Among these, dynamical response of rotor-bearing system is an important research topic, the amplitudes of transient displacements and loads are the main concern parameters, and influences due to different parameters such as internal damping, gyroscopic moment and support stiffness are analyzed. In aspects of mechanical mechanism, Genta[9], Dzenan[10], Kalinowski[11], Li[12]etc. studied the sudden unbalance response by derivation formulas based on mechanical models. They demonstrated the sudden unbalance could result in amplification of vibration response (shock effect), which related to the operating state of rotor. In the view of mechanism testing, Hibner[13], Sakata[14], Kalinowski[11], Li[15] etc. built the rotor systems and analyzed the transient response characteristics and nonlinear dynamical behaviors based on the test data.

Most test rigs in current literatures are comprised of rotor with disks between bearings and simple support systems. However, the low pressure rotor system with large fan blade is usually overhung form in the high bypass ratio aero engines. The mass and moment of inertia primary of rotor system locate at the cantilever end, so the gyroscopic effects could not be ignored. Thus, the rotor system mentioned above is usually considered as a typical flexible rotor system operating above several critical speeds. In addition, the supporting system is not rigid, and the loads are transmitted to mount through a series of flexible components. Both structural and mechanical characteristics should be considered in the research. In this paper, an overhung rotor system with flexible support is built, and the responses of rotor under different conditions (subcritical and supercritical) are tested and analyzed, then the impact effects are compared with the mechanical models. Finally the dynamical characteristics of overhung rotor due to sudden unbalance are summarized and discussed.

# 1 EXPERIMENTAL SCHEME

## 1.1 Overall structure of the test rig

Figure 1 and Figure 2 show the photo and schematic diagram of the “overhung rotor-support system” test rig respectively, which is mainly composed of drive and control system, testing rotor system and vibration test system.



Figure 1 Overhung rotor-support test rig

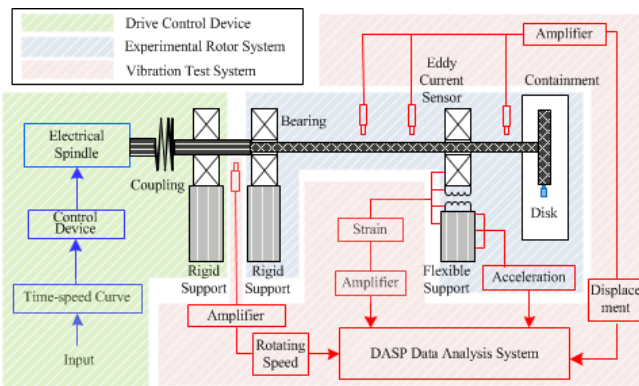


Figure 2 Schematic diagram of test system

The drive control device is driven by high-speed electrical spindle, which can rotate according to a predetermined time-rotating speed curve. The coupling is a rubber sleeve elastic coupling which would not induce other exciting force.

The rotor is a single disc overhung support rotor, with a rigid support and an elastic support frame. At the elastic support frame, the load transmitting path is squirrel cage - inner casing - stator plate - outer casing - flange. The bearings are ball bearings lubricated and cooled by oil.

Vibration test system is used to obtain time and frequency domain features of structural components’ vibration response. Eddy current sensors and laser sensors have been arranged on the rotor shaft to obtain the rotor vibration displacement response. Strain gauges and acceleration sensors have been arranged on the stator support parts to obtain the strain and the vibration acceleration of the support parts.

## 1.2 Sudden unbalance device

There are two functions essential for the test rig to undertake sudden unbalance test during multi-states: (1) Timing controllability of sudden unbalance; (2) Unbalance eccentricity can be compensated, so as to ensure the system safely pass through the critical speed at a low unbalanced load

during deceleration process.

Figure 3 shows the structure of sudden unbalance device. The design idea of the device comes from literature [11], it consists of two independent controlled and independent triggered spring rod, the unbalanced mass is cut by the movement of hammer in the horizontal direction. The function of the sudden unbalance device is as follows:

(1) At a predetermined speed, both spring rods is pushed right that it would not touch the disk.

(2) The hammer is moved immediately by the spring to the right. After shearing-off the mass on the one side of the disk the rotor generate a sudden imbalance force.

(3) When the rotor under large unbalance needs to run down through the critical speed, the other spring rod should be pulled left, and the mass at 180° phase apart is cut to compensate unbalance eccentricity.

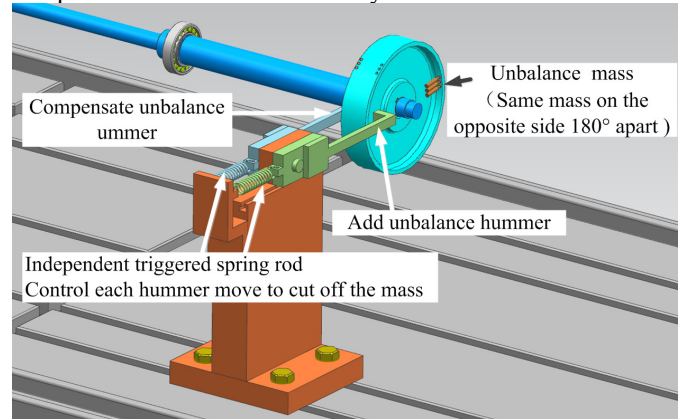


Figure 3 Sudden unbalance device

## 1.3 Measuring points arrangement

The arrangement of measuring points is shown in Figure 4. Eddy current sensors, strain gauges and acceleration sensors are arranged on the load-transmitting path. They can test the rotor vibration response under sudden unbalance, including the rotor shaft orbit, rotor deformation elastic line, stator vibration response of the system.

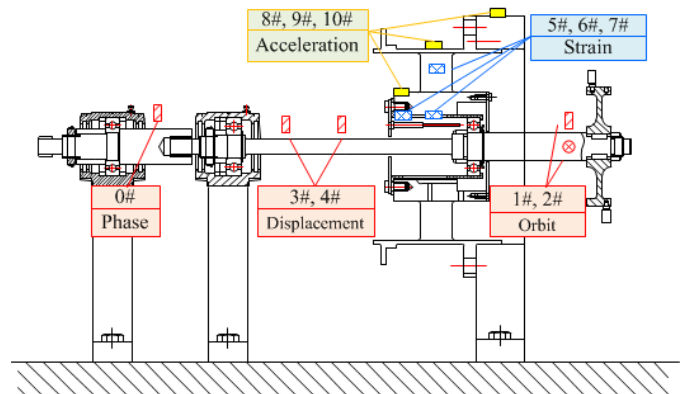


Figure 4 Measuring points on test rig

Five eddy current sensors of Bentley 3300XL (providing up to 2 mm of linear range with an output of 200 mV/mil) are placed on the rotor. Among them, the 0# measuring point corresponds to the phase signal of the rotational speed. 1 #

and 2 # measuring points locate at the overhung sections and in the horizontal and vertical direction respectively, and the orbit of rotor could be picked by them. The displacements of the shaft between the two supports are measured at 3# and 4# measuring points.

Three strain gauges (2.06mV/V) at 5~7# measuring point, with 5 # and 6 # located on squirrel cage bars, and 7# located at support plate on the support frame.

Three acceleration sensors of PCB-352B10 (10.68 mV/g, ±4905 m/s<sup>2</sup>,1-17000Hz) named 8~10# locate at inner case, outer case of support frame and the flange respectively.

## 2 NATURAL CHARACTERISTICS TEST

Before the sudden unbalance test, the test on the natural characteristics of test system is carried out firstly, including the rotor critical speed and the modal frequencies of stator support structure.

The 0→6000rpm→0 process displacement response testing was carried out for the rotor system, and the Bode plot is shown in Figure 5. As a result, the rotor critical speed is 4208rpm by testing.

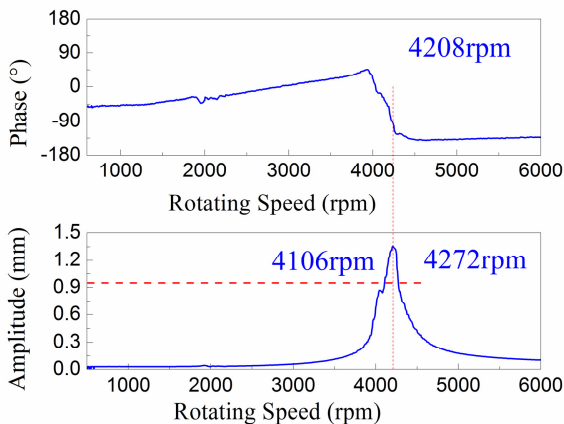


Figure 5 Bode plot of the rotor system

For the stator support frame, hammer excitation method was applied, and the first two natural modal frequencies for the casing are 775Hz and 1622Hz, which are much higher than the operating frequency and its multiple frequency, thus no coupled vibrations occurred during the test.

The damping of the system is estimated by half power point method, the critical speed and the speeds in half power points are marked in the bode plot. Finally, the damping ratio, which will be used in the numerical simulation, turns out to be 0.02.

## 3 SUDDEN UNBALANCE RESPONSE OF THE SUBCRITICAL ROTOR

According to the testing critical speed, subcritical rotational speed was chosen as 3000rpm, and unbalance mass was 10g, so the unbalance load is 74.02N.

### 3.1 Orbits of rotor

1s before the sudden unbalance until 2s after the unbalance,

during this period, the response of rotor system axis orbit are shown in Figure 6, the phase of sudden unbalance mass was 0°

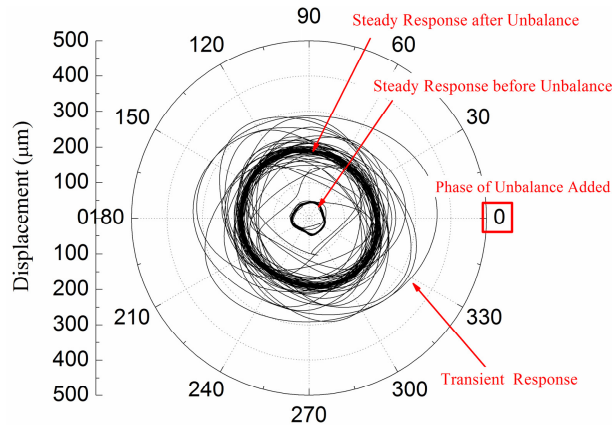


Figure 6 Measured rotor orbit at subcritical state

Figure 6 shows that, during the steady-state process, both before and after adding the unbalance force, the rotor axis orbit is basically circular, and the amplitude of the vibration response increase due to the increased unbalance. At the moment the unbalance is added, there is a sudden increase of the amplitude of rotor vibration lasting for several cycles, then attenuating and tending to be stable. The transient response orbit of the rotor presents as elliptical shape, and transient response amplitude in the unbalance mass phase direction (0° ) is maximum, while vertical to the unbalance mass phase direction (90° ), the amplitude is relatively smaller.

### 3.2 Time / frequency response characteristics

Figure 7 shows the rotor vibration response along the sudden unbalance phase direction. The impact factor R [11] can be used to quantitatively assess the shock effect of the rotor during sudden unbalance process. R is defined as the ratio between the maximum response and the steady-state amplitude after sudden unbalance, i.e.:

$$R = f_{\max} / f_s \tag{1}$$

To consider system initial unbalance amplitude  $f_i$ , equation (1) is revised as:

$$R = (f_{\max} - f_i) / (f_s - f_i) \tag{2}$$

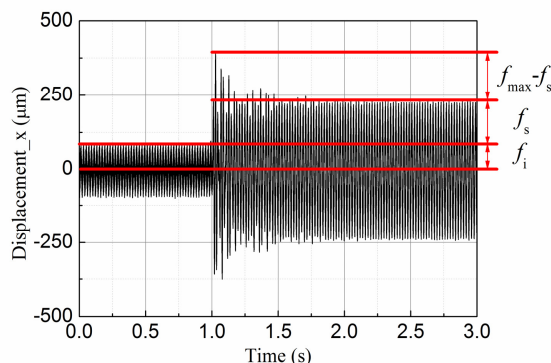
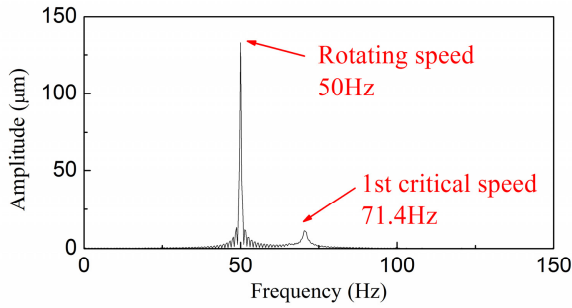


Figure 7 Time history response in x direction at subcritical state

According to Figure 7, the impact factor for subcritical state rotor in the unbalance phase direction is 2.17, while vertical to the unbalance phase direction, the factor value is 1.73.



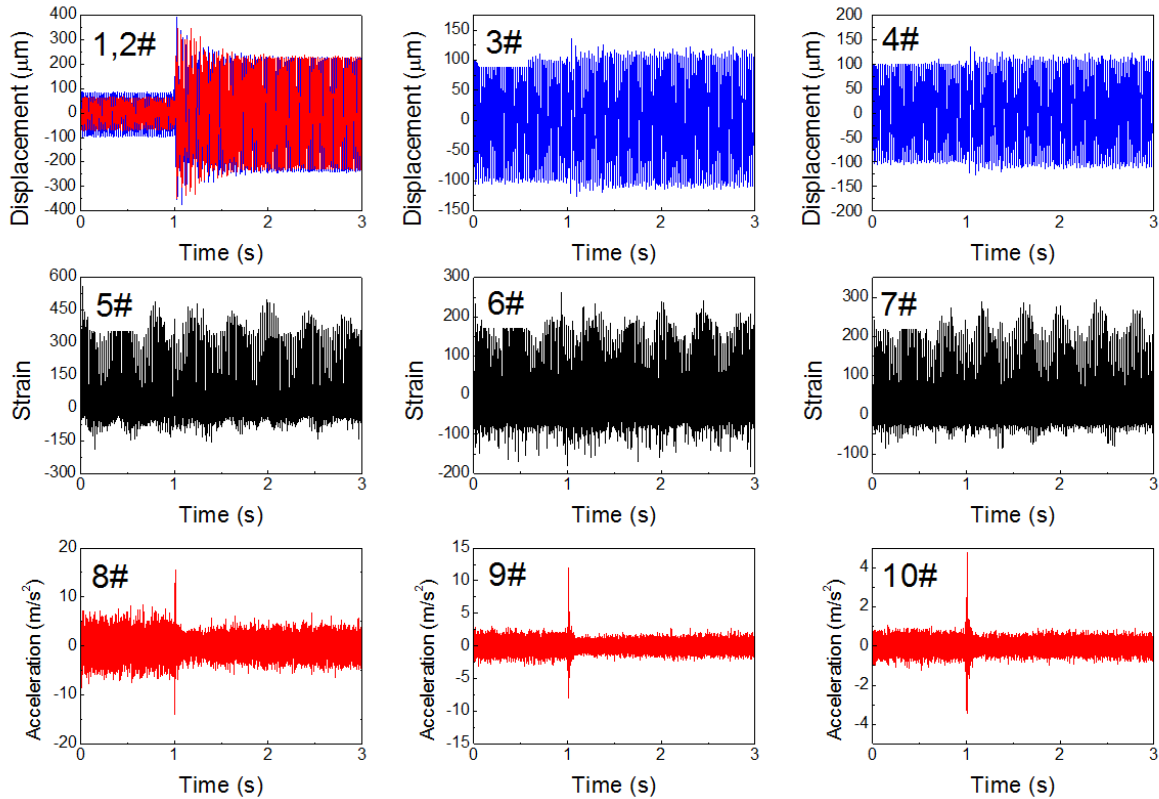
**Figure 8 The response in x direction at subcritical state in frequency domain**

During this period of 1s before and after the sudden unbalance, the rotor frequency response is shown in Figure 8. Frequency of rotor critical speed (4284rpm  $\approx$  4208rpm, 71.4Hz) exists in the above plot except for the frequency of the rotor speed, indicating that sudden unbalance load

aroused natural mode vibration of the rotor.

### 3.3 Responses of other points

Analyze on the other measuring points along the load transmitting path, Figure 9 shows the time domain response for 2#~10# measuring points. 2#, 3# and 4# measuring points correspond to the rotor vibration response. It's obvious that there is a evident impact effect at the rotor's overhang segment (2#), while for the shaft segment between the fulcrums (3# and 4#), the vibration response is less affected by the unbalance value. At 5#, 6# and 7# measuring points, there are strain gauges located at squirrel cage and stator support plate, and the structural strain response changes little after sudden unbalance. There are acceleration sensors at braced frame related to 8#, 9# and 10# measuring points, the impact effect during the sudden unbalance is evident, but the influence on steady state vibration amplitude is relative small, the main reason is that the unbalance force is relatively small at low rotation speed.



**Figure 9 Time history response by all sensors during subcritical test**

## 4 SUDDEN UNBALANCE RESPONSE OF THE SUPERCRITICAL ROTOR

Sudden unbalance test was carried out under the situation that rotor operated at supercritical speed (6000rpm), with 10g unbalance mass which is equal to 296.08N corresponding unbalance load.

### 4.1 Orbits and time / frequency response characteristics

The measured orbit is shown in Figure 10, the characteristics of rotor vibration response is basically the same as rotor in subcritical state, the difference is that, for the peak values of rotor vibration response, the phase is not at the unbalance mass phase ( $0^\circ$ ), but vertical to that direction

(90°). The impact factors at the two directions are 2.79 and 3.66 respectively, which are bigger than those in subcritical condition, indicating that the flexible rotor are more sensitive to sudden unbalance excitation.

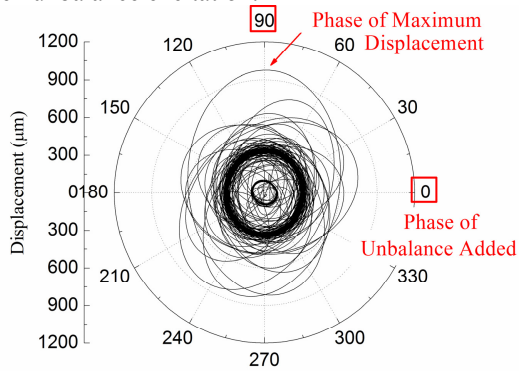


Figure 10 Measured rotor orbit at supercritical state

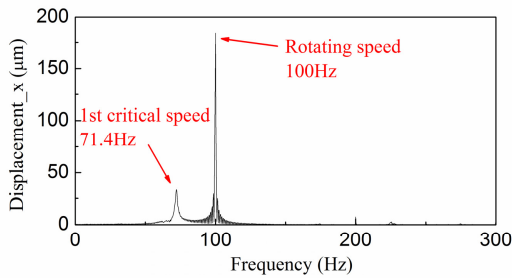


Figure 11 The response in x direction at supercritical state in frequency domain

The frequency response during the period 1s before and after the unbalance excitation is shown in Figure 10, similar with the response under subcritical state, the frequency ingredients include the rotating speed and the critical speed, and the proportion of the amplitude of critical speed frequency to rotating speed frequency is relatively higher (see Figure 11), which also indicates that the rotor shock effect is more obvious.

#### 4.2 Responses of other points

Analyze on the measuring points along the load transmitting path as above, and the time domain response result is in Figure 12. For the rotor vibration displacement (2#, 3# and 4# measuring point), the response also shows the local effects of overhung rotor due to sudden unbalance which is the same as in the subcritical state. For the rotor strain signals (5# and 6#) at the squirrel cage, the impact effect is unobvious at the moment the sudden unbalance applied, but the vibration response amplitudes are different before and after the sudden unbalance, indicating that load increased significantly as the unbalance amount increased. There is no obvious response at the supporting plate (7#) for its small deformation. It's worth noticing that the acceleration response is interesting for the braced frame (8#, 9# and 10#): when the unbalance value increases, the amplitude of the response reduces conversely, and this phenomenon has been reproduced in repeated tests. The phenomenon needs more theoretical analysis, numerical simulation and in-depth tests analysis to explain.

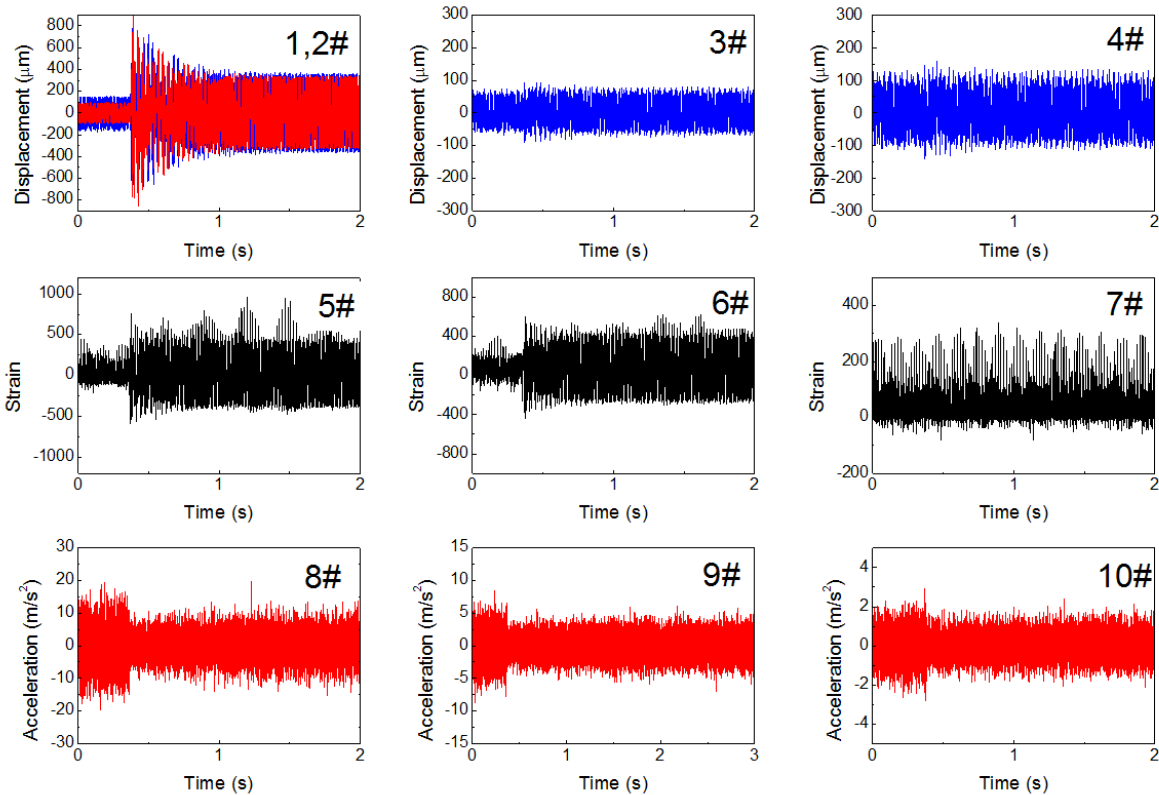


Figure 12 Time history response by all sensors during supercritical test

## 5 MECHANISM ANALYSIS

The loads on the rotor due to sudden unbalance could be easily described by considering the unbalance force as variable in time. The equation of motion is Equation (3), in which the load on the right is unbalance force multiplied by Dirichlet function  $H(t)$ .

$$M\ddot{q} + (C - \omega G)\dot{q} + Kq = H(t)F \quad (3)$$

$$\begin{cases} H(t) = 0 & \text{when } t < 0 \\ H(t) = 1 & \text{when } t \geq 0 \end{cases} \quad (4)$$

An overhung rotor based on the test rotor is taken as the example to analyze the response characteristics of sudden unbalance. The structure of rotor considering the distribution of stiffness, mass and gyroscopic moment is shown in Figure 13, and the parameters are listed in Table 1. The derivation of the fully system of ordinary differential equations can be derived by finite element method.

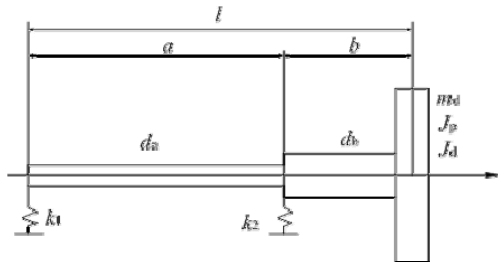
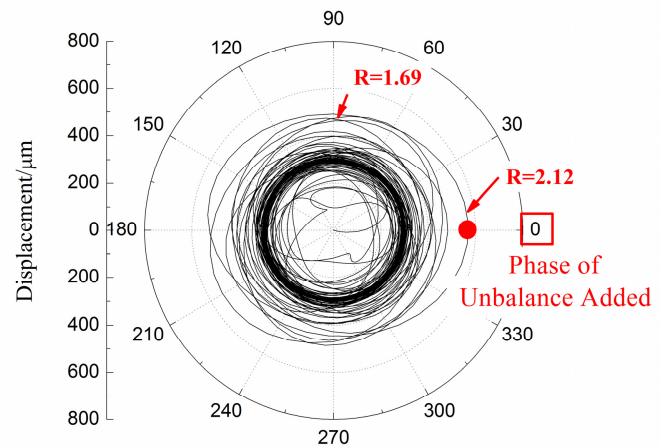


Figure 13 Mechanical model of overhung rotor

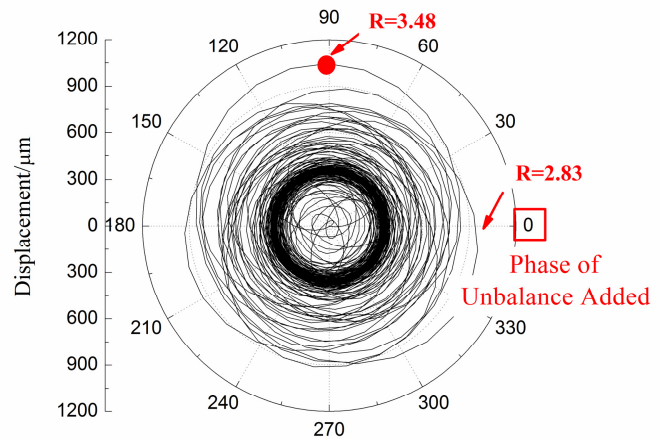
Table 1 Parameters in mechanical model

Parameters	Value	Unit
Density	7800	kg/m <sup>3</sup>
Elasticity modulus	210	GPa
Poisson's ratio	0.3	-
Length of rotor	0.45	m
Length of shaft a	0.15	m
Length of shaft b	0.30	m
Diameter of shaft a	0.035	m
Diameter of shaft b	0.020	m
Stiffness of bearing 1	1×10 <sup>8</sup>	N/m
Stiffness of bearing 2	4×10 <sup>6</sup>	N/m
Mass of disk	1.73	kg
Polar moment of inertia	4.8×10 <sup>-3</sup>	kg·m <sup>2</sup>
Damping ratio	0.02	-
Sudden unbalance	75	g·cm

The vibration equation is solved by Newmark's method of direct integration based on MATLAB. The orbits of the rotor in different rotating speeds are shown in Figure 14. The orbits during transient process appear to be ellipse shape, and the maximum are in different phase position. The impact factors are also shown in the figures.



(a) Subcritical state (3000rpm)



(b) Supercritical state (6000rpm)

Figure 14 Orbits of Jeffcott rotor due to sudden unbalance

Comparison of impact factors between the theoretical and experimental results is presented at Table 2. The results are basically satisfying compared with each other both in the subcritical state and the supercritical state. The errors are less than 3%. The results show that the mechanical model could capture the transient dynamic characteristics of rotor system due to sudden unbalance.

Table 2 Comparison of the impact factor R

State	Phase	Model	Test	Error (%)
Subcritical	0°	2.12	2.17	2.36
	90°	1.69	1.73	2.37
Supercritical	0°	2.83	2.79	1.41
	90°	3.56	3.66	2.73

## 6 CONCLUSION AND OUTLOOK

(1) Sudden unbalance load brings impact effects on the rotor. The amplitude peak of vibration response of the rigid rotor is located at the unbalance phase, while for flexible rotor, the phase of the peak response is located at 90° instead of the unbalance phase. The frequency domain responses contain rotational speed frequency and rotors low-speed natural frequency.

(2) The impact effect of the rotor system due to sudden unbalance can be evaluated by the impact factor. Test results show that the impact effect is more evident for flexible rotor. For the safety design of typical flexible rotor system (such as low pressure rotor system of high bypass ratio turbofan engine), the structure and mechanical characteristics should be well considered.

(3) For the overhang rotor system, when the sudden unbalance load is applied to the disk at overhang segment, the vibration response mainly affects the overhang segment vibration, instead of the whole rotor, that is to say, the vibration response exhibits local effect.

In this article, the research focuses on studying the characteristic of rotor system vibration response under sudden unbalance excitation, and quantitatively assessing the impact effect. The influence due to the rotating speed should be tested in the further study. Actually the blade loss event happening in rotating machinery contains, in engineering practice, quite complex mechanical problems including severe rubbing deceleration process, asymmetric rotor and so on. These problems need more testing analysis, combining with theoretical and numerical study, to support the structural safety design for high bypass ratio turbofan engines.

## NOMENCLATURE

$f_i$	Steady amplitude under initial unbalance
$f_s$	Steady amplitude under changed unbalance
$f_{\max}$	Maximum response due to sudden unbalance
$H(t)$	Dirichlet function
$x_0$	Steady state amplitude of the unbalance response
$F$	Exciting force
$M, C, K$	Matrixes of mass, stiffness and damping
$q, \dot{q}, \ddot{q}$	Vectors of displacement, velocity, acceleration
$R$	Impact factor

## ACKNOWLEDGEMENTS

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